

Creating the Universe from Brane-Antibrane Annihilation

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(Dated: 22 March 2004)

When p -dimensional branes annihilate with antibranes in the early universe, as in brane-antibrane inflation, stable $(p-2)$ -dimensional branes can appear in the final state. We reexamine the possibility that one of these $(p-2)$ -branes could be our universe. In the low energy effective theory, the final state branes are cosmic string defects of the complex tachyon field which describes the instability of the initial state. We quantify the dynamics of formation of these vortices. This information is then used to estimate the production of massless gauge bosons on the final branes, due to their coupling to the time-dependent tachyon background, which would provide a mechanism for reheating after inflation. We improve upon previous estimates indicating that this can be an efficient reheating mechanism for observers on the brane.

PACS numbers: 98.80.Cq

I. INTRODUCTION

In the last few years significant progress has been made in constructing string theoretic cosmological models where inflation is driven by the naturally occurring potentials between D-branes and their antibranes [1]-[3]. The formation of lower-dimensional branes at the end of inflation can lead to interesting signals: cosmic-string-like or higher dimensional defects could be observable remnants [4], possibly providing a rare clue to the stringy origin of inflation. A more radical idea was explored in [5]: perhaps our own observable universe is such a defect in the higher-dimensional spacetime predicted by string theory. Since the stable branes in Type IIB string theory have spatial dimensionalities which are odd, a 3-brane would have descended from annihilation of 5-branes in this picture, and thus they would be codimension-two defects in of the effective 6D theory. Codimension-two braneworlds have attracted interest lately because of their novel features, which might have some bearing on the cosmological constant problem [6, 7].

Our interest in this scenario is motivated by questions about the efficiency of reheating in brane-antibrane inflation [8]. It is possible that the energy density liberated from the brane collisions will be converted mostly into closed string states, ultimately gravitons, and not necessarily into visible radiation [9]. A generic mechanism which could avoid this problem was proposed in [5], wherein the reheating in D-brane driven inflation is due to the coupling of massless gauge fields to a time-dependent tachyon condensate, which describes the annihilation process. However, ref. [5] considered only the formation of a tachyon kink instead of the more realistic case of a vortex, and it used a somewhat crude ansatz for the background tachyon field. The problem of finding the actual tachyon background predicted by string theory was studied numerically in [10] but no attempt was made to improve on the reheating computation. In this paper

we aim to analytically determine the dynamics of formation of lower dimensional branes described as tachyon defects—both kinks and vortices—and to improve on the reheating calculation of [5].

Let us begin by describing the scenario we have in mind. In the simplest version of D-brane inflation a parallel brane and antibrane begin with some separation between them in one of the extra dimensions. Although parallel branes are supersymmetric and have no force between them, the brane-antibrane system breaks supersymmetry so that there is an attractive force and hence a nonvanishing potential energy. It is the latter which drives inflation. Once the branes have reached a critical separation one of the stretched string modes between the branes, T , becomes tachyonic and the branes become unstable to annihilation. The tachyon field starts from the unstable maximum $T = 0$ and rolls towards the vacuum $T \rightarrow \pm\infty$. However, topological defects may form through the Kibble mechanism [4] so that $T = 0$ stays fixed at the core of the defect. These defects are known to be consistent descriptions of branes whose dimension is lower than that of the original branes [11, 12]. For example, the brane-antibrane system has a complex tachyon field, leading to vortices which represent codimension-two branes. On the other hand, an unstable brane has a real tachyon which leads to kinks representing codimension-one branes.

The formation of tachyon defects at the endpoint of D-brane inflation is a dynamical process where the tachyon couples to gauge fields which will be localised on the descendant brane. It is thus expected that some radiation will be produced by the rolling of the tachyon and the problem of reheating becomes quantitative: can this effect be efficient enough to strongly deplete the energy density of the tachyon fluid so the the universe starts out being dominated by radiation rather than cold dark matter? It is important to stress that though the situation is somewhat analogous to that of hybrid inflation (where the tachyon plays the role of the unstable direction in

field space which allows for inflation to end quickly) the mechanism for reheating is qualitatively different. The difference is that in the low energy effective field theory which describes the tachyon T , the potential is minimized at $T = \pm\infty$ and there are no oscillations about the minimum of the potential. In a normal hybrid inflation model, T would have a minimum at some finite value and the oscillations of T around its minimum would give rise to reheating in the usual way. In the present case, the time dependence of the background is monotonic, not oscillatory. Reheating thus might seem to resemble gravitational particle production [13] rather than the standard picture in which the inflaton decays. However, in this work we highlight an important difference between reheating through tachyon condensation and gravitational particle production, which can make the former much more efficient: there is a divergence in the stress-energy tensor of the tachyon field within a finite time, which corresponds to the formation of the lower-dimension D-brane.

In this paper we study analytically the dynamical formation of the tachyon vortex and improve the reheating calculation, using a slightly simplified model of particle production by tachyon condensation, which captures the essential physics revealed by the analysis of vortex formation. In section II we review the formation of tachyon kinks which describes the condensation of a brane to a brane of codimension-one. In sections III-V we study analytically the formation of a tachyon vortex on the brane-antibrane pair. Section III introduces Sen's action for the complex tachyon field describing this situation. Section IV presents new analytic results for the time-dependent, complex tachyon field representing vortex formation, both near to and far from the vortex core. In section V we show that the stress-energy tensor for the system splits into a localized, singular piece describing the descendant branes, plus a bulk contribution that describes the rolling tachyon condensate. Section VI introduces the effective action for U(1) gauge bosons which become localized on the final-state 3-brane, in the rolling tachyon background. This provides a model for the visible radiation produced during reheating. In section VII we calculate the energy density of this produced radiation on the 3-brane, using some reasonable simplifying assumptions. Section VIII gives our conclusions, including speculation about how the final brane-antibrane system could be stabilized.

II. DYNAMICAL TACHYON KINK FORMATION ON UNSTABLE DP-BRANES

In this section we review the dynamical formation of a $D(p-1)$ -brane through tachyon condensation on an unstable Dp -brane, and derive a few new results. The equations of motion in this case are simpler than in the case of the vortex and we will use the analysis of this section to reinforce our conclusions when we analyze the

vortex since many of the results are quite analogous.

A. Effective Field Theory and Equations of Motion

We will work with the effective action for the tachyon on an unstable Dp -brane [14, 15]¹

$$S = - \int V(T) \sqrt{-\det[\eta_{MN} + \partial_M T \partial_N T]} d^{p+1}x \quad (1)$$

where we have set the gauge fields and transverse scalars to zero. We use the potential $V(T) = \tau_p \exp(-T^2/a^2)$ where τ_p is the tension of a Dp -brane and $a = 2\sqrt{\pi\alpha'}$. The value of the constant a is chosen so that the potential satisfies the normalization condition

$$\int_{-\infty}^{+\infty} V(y) dy = 2\pi\sqrt{\alpha'}\tau_p = \tau_{p-1} \quad (2)$$

proposed in [11]. This normalization was used in [11] to fix the tension of the singular static kink solution of the action (1) to correspond to the tension of a $D(p-1)$ -brane. For a time-dependent kink solution we take T to be a function of $x^\mu = (t, x)$ so that the action (1) becomes

$$S = - \int V(T) \sqrt{1 + \partial_\mu T \partial^\mu T} d^{p+1}x. \quad (3)$$

Static solutions of the theory (3) are well studied in the literature [11, 16]. Inhomogeneous solutions have also been studied in some detail [5, 10, 17, 18]. The energy momentum tensor for (3) is

$$T_{\mu\nu} = \frac{V(T)}{\sqrt{1 + \partial_\rho T \partial^\rho T}} \partial_\mu T \partial_\nu T - \eta_{\mu\nu} V(T) \sqrt{1 + \partial_\rho T \partial^\rho T}, \quad (4)$$

and the Euler-Lagrange equation of motion is

$$\partial^\mu \partial_\mu T - \frac{\partial_\mu \partial_\nu T \partial^\mu T \partial^\nu T}{1 + \partial^\rho T \partial_\rho T} - \frac{V'(T)}{V(T)} = 0 \quad (5)$$

where $V'(T) = \frac{\partial V(T)}{\partial T}$. It is worth noting, as in [19], that the equation of motion (5) is equivalent to conservation of energy $\partial_\mu T^{\mu\nu} = 0$ for nonconstant T since $\partial_\mu T^{\mu\nu} = \partial^\nu T \left[\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu T)} \right) - \frac{\partial \mathcal{L}}{\partial T} \right]$. It will be useful in the ensuing analysis to define

$$\Sigma = \frac{V(T)}{\sqrt{1 + \partial_\mu T \partial^\mu T}}. \quad (6)$$

¹ The convention for indices is that upper case roman indices $\{M, N\}$ run over the full space-time coordinates $\{0, 1, \dots, p\}$, greek indices $\{\mu, \nu\}$ run over the defect coordinates $\{0, 1\}$ and “hatted” greek indices $\{\hat{\mu}, \hat{\nu}\}$ run over the remaining spatial coordinates $\{2, 3, \dots, p\}$. We use metric signature $\text{diag}(-1, 1, 1, \dots)$.

B. Solutions Near the Core of the Defect

At the core of the kink we expect the field to stay pinned at $T = 0$. Consider initial data $T(t = 0, x) = T_i(x)$ and $\dot{T}(t = 0, x) = \dot{T}_i(x) = 0$ ². One expects the field to start to roll where $T_i(x) \neq 0$ due to the small displacement from the unstable maximum $V'(T) = 0$. At $t = 0$ the equation of motion (5) is

$$\ddot{T}_i(x)(1 + T_i'(x)^2) = T_i''(x) + 2a^{-2}T_i(x)(1 + T_i'(x)^2).$$

Clearly any point x_0 where $T_i(x_0) = T_i''(x_0) = 0$ will be a fixed point where $\ddot{T}(t, x_0) = 0 = \dot{T}(t, x_0)$ throughout the evolution. We restrict ourselves only to considering initial data such that $\text{sgn}(\dot{T}_i(x)) = \text{sgn}(T_i(x))$ for all x to ensure that the solutions are increasing.

At the site of the kink (which we take to be $x_0 = 0$) we have $T = 0$; hence there should always be some neighbourhood of the point $x = 0$ where we can take $V'(T) \cong 0$ so that (5) yields

$$\ddot{T}(1 + T'^2) = (1 - \dot{T}^2)T'' + 2\dot{T}T'\dot{T}'. \quad (7)$$

This has an increasing solution with $T'' = 0$

$$T(x, t) = x \tan \left[\frac{\omega}{a}(t - t_c) + \frac{\pi}{2} \right]. \quad (8)$$

Near the site of the kink the slope of the tachyon field diverges as $(t_c - t)^{-1}$ as t approaches the critical time t_c , similar to the solutions in [10].

The finite-time slope divergence was observed both numerically and analytically in [10] and leads to the formation of a singularity in the energy density at $t = t_c$. This effect was also found in an exact string theoretic calculation in [20]. As $t \rightarrow t_c$ we have $\Sigma(t \rightarrow t_c, x \cong 0) \rightarrow 0$. The case $\Sigma = \text{const}$ arises as a first integral of the motion in the static case and the limit $\Sigma \rightarrow 0$ corresponds to the singular soliton solution of Sen [16]. It is natural to expect, then, that as $t \rightarrow t_c$, near $x = 0$, the tachyon field $T(t = t_c, x = 0)$ coincides with the stable kink solution of Sen and the time evolution in this neighbourhood stops. We will argue that at this point a codimension-one brane has formed.

C. Vacuum Solutions

Away from the site of the kink the field is expected to roll towards the vacuum $T \rightarrow \pm\infty$ so that $V(T) \rightarrow 0$ at late times for $x \neq 0$. To analytically study the dynamics near the vacuum it is easiest to work in the Hamiltonian formalism [21, 22] since the Lagrangian vanishes in the limit $V(T) \rightarrow 0$, whereas the Hamiltonian

remains well-defined. Defining the momentum conjugate to T as $\Pi = \delta S / \delta \dot{T}$ the Hamiltonian is given by $\mathcal{H} = \sqrt{\Pi^2 + V(T)^2} \sqrt{1 + \partial_i T \partial_i T}$. It is useful to rewrite Hamilton's equations of motion, $\dot{\Pi} = -\frac{\partial \mathcal{H}}{\partial T}$ and $\dot{T} = \frac{\partial \mathcal{H}}{\partial \Pi}$, in a manifestly covariant way as

$$\partial^\mu T \partial_\mu T + 1 = \frac{V(T)^2}{\Sigma^2} \quad (9)$$

and

$$\Sigma \partial_\mu [\Sigma \partial^\mu T] = V(T) V'(T), \quad (10)$$

where $\Sigma = \Pi / \dot{T}$ is defined in (6). In the limit $V(T) \rightarrow 0$ equations (9), (10) yield

$$\partial^\mu T \partial_\mu T + 1 = 0 \quad (11)$$

and

$$\partial_\mu [\Sigma \partial^\mu T] = 0. \quad (12)$$

The solutions of (11) were found for arbitrary Cauchy data in [17] using the method of characteristics. The generic solutions exhibit the formation of caustics where second and higher order derivatives become singular. Caustics are known to form in systems with a pressureless fluid, which is a good description of the tachyon field as it approaches its ground state $T \rightarrow \infty$. It is not known whether the caustics are a genuine prediction of string theory or just an artifact of the derivative truncation which leads to the Born-Infeld Lagrangian for the tachyon.

In any case, caustics are not present in the simplest solution of eq. (11), $T = \pm t$, which is the asymptotic form for the homogeneously rolling tachyon. For $\dot{T}^2 = 1$, eq. (12), which is equivalent to energy conservation, implies that $\Sigma(t, x) = \Sigma(x)$ is an arbitrary function of x . In this regime the energy momentum tensor (4) is identical to that of pressureless dust $T_{\mu\nu} = \Sigma(x) u_\mu u_\nu$ where $u_\mu = \partial_\mu T$ is interpreted as the local velocity vector and Σ is interpreted as a Lorentz-invariant matter density [15, 21, 22].

D. Stress-Energy Tensor

We are interested in the behavior of T_{MN} as $t \rightarrow t_c$. First we consider the neighbourhood near $x = 0$ where $T(t, x) \cong kx / (t_c - t)$ as $t \rightarrow t_c$ (see 8). Near $x = 0$, the Hamiltonian is

$$T_{00} \cong \frac{\tau_p k}{t_c - t} \exp \left(-\frac{k^2 x^2}{a^2 (t_c - t)^2} \right)$$

and

$$\lim_{t \rightarrow t_c^-} T_{00} = \sqrt{\pi} a \tau_p \delta(x) = \tau_{p-1} \delta(x) \quad (13)$$

² For the remainder of this section the dot denotes differentiation with respect to time while the prime denotes differentiation with respect to the x coordinate.

using the normalization (2) for the potential. Similarly

$$T_{\hat{\mu}\hat{\nu}} \rightarrow -\tau_{p-1} \delta(x) \delta_{\hat{\mu}\hat{\nu}}$$

and $T_{11} \rightarrow 0$ as $x \rightarrow 0$.

Consider now the late-time behavior of T_{MN} away from the site of the kink. Using the solutions of section II C we find $T_{00} \rightarrow \Sigma(x)$ while T_{11} , T_{01} and $T_{\hat{\mu}\hat{\nu}}$ tend to zero for $x \neq 0$.

To summarize, we find that in the limit of condensation the energy momentum tensor is identical to that of a $D(p-1)$ -brane:

$$\begin{aligned} T_{00} &= \tau_{p-1} \delta(x) + \Sigma(x) \\ T_{11} &= T_{01} = 0 \\ T_{\hat{\mu}\hat{\nu}} &= -\tau_{p-1} \delta(x) \delta_{\hat{\mu}\hat{\nu}}. \end{aligned}$$

The extra bulk energy density $\Sigma(x)$ is similar to the result in [19] and corresponds to what has been dubbed tachyon matter.

III. EFFECTIVE TACHYON FIELD THEORY ON THE BRANE-ANTIBRANE PAIR

We would like to generalize the results of the previous section to study the dynamical formation of a tachyon vortex and hence a codimension-two brane. This is the more realistic situation, since the stable D-branes of a given string theory are those whose dimensions differ by multiples of two. We will work with an effective action proposed by Sen [11] for the tachyon on a brane-antibrane pair. The field content for this system is a complex tachyon field T , massless gauge fields $A_\mu^{(1)}$, $A_\mu^{(2)}$ and scalar fields $Y_{(1)}^I$, $Y_{(2)}^I$ corresponding to the transverse fluctuations of the branes. The index $(i) = (1), (2)$, which we call the brane index, labels which of the original branes (actually the brane or the antibrane) the field is associated with. The effective action is:

$$\begin{aligned} S = - \int V(T, Y_{(1)}^I - Y_{(2)}^I) & \left(\sqrt{-\det M^{(1)}} \right. \\ & \left. + \sqrt{-\det M^{(2)}} \right) d^{p+1}x \end{aligned} \quad (14)$$

where ³

$$\begin{aligned} M_{MN}^{(i)} &= g_{MN} + \alpha' F_{MN}^{(i)} + \partial_M Y_{(i)}^I \partial_N Y_{(i)}^I \\ &+ \frac{1}{2} D_M T D_N T^* + \frac{1}{2} D_M T^* D_N T, \end{aligned} \quad (15)$$

³ Greek indices $\{\mu, \nu\}$ are now understood to run over the coordinates $\{0, 1, 2\}$ on which the vortex solutions depend, and hatted greek indices $\{\hat{\mu}, \hat{\nu}\}$ run over the spatial coordinates parallel to the vortex $\{2, 3, \dots, p\}$, where $p = 6$ for a vortex which describes a 3-brane. Upper case roman indices $\{M, N\}$ still run over the full space-time coordinates $\{0, 1, \dots, p\}$. Finally it will be convenient later on to refer to lower case roman indices $\{m, n\}$ which run over only the time and radial coordinates $\{0, 1\}$.

$$F_{MN}^{(i)} = \partial_M A_N^{(i)} - \partial_N A_M^{(i)}, \quad D_M = \partial_M - iA_M^{(1)} + iA_M^{(2)}. \quad (16)$$

For the remainder of this paper we will ignore the transverse scalars and choose $V(T, 0) = V(T) = \tau_p \exp(-|T|^2/a^2)$ where a is chosen so that the static singular vortex solutions of the theory (14) have the correct tension to be interpreted as codimension 2 D-branes according to the normalization proposed in [11]. We will discuss the normalization of the potential proposed in [11] in more detail when we calculate the energy momentum tensor for the theory (14).

Though the action (14) was not derived from first principles it obeys several necessary consistency conditions which are discussed in [11]. There have been various other proposals for the tachyon effective action and vortex solutions on the brane-antibrane pair [23, 24, 25]. See [26] for a discussion of various models including (14).

IV. VORTEX SOLUTIONS ON THE BRANE-ANTIBRANE PAIR

To construct vortex solutions we ignore the transverse scalars and take the remaining fields to depend only on the polar coordinates $x^\mu = (x^0, x^1, x^2) = (t, r, \theta)$ with metric $g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dr^2 + r^2 d\theta^2$. Since the vortex solution should have azimuthal symmetry we make the ansatz:

$$T(t, r, \theta) = e^{i\theta} f(t, r), \quad A_\theta^{(1)} = -A_\theta^{(2)} = \frac{1}{2} g(t, r) \quad (17)$$

with all other components of $A_\mu^{(i)}$ vanishing. This generalizes the ansatz used in [11] to include time-dependence in the fields f and g . For (17) one has

$$D_t T = e^{i\theta} \dot{f}(t, r), \quad D_r T = e^{i\theta} f'(t, r),$$

$$D_\theta T = e^{i\theta} i(1 - g(t, r)) f(t, r)$$

and

$$F_{t\theta}^{(1)} = \frac{1}{2} \dot{g}(t, r) = -F_{t\theta}^{(2)}, \quad F_{r\theta}^{(1)} = \frac{1}{2} g'(t, r) = -F_{r\theta}^{(2)}$$

where the dot denotes differentiation with respect to time and the prime now denotes differentiation with respect to the radial coordinate. The matrices $M_{MN}^{(i)}$ are

$$\begin{aligned} [M_{MN}^{(1)}] &= \begin{bmatrix} -1 + \dot{f}^2 & \dot{f}f' & \alpha' \dot{g}/2 & 0 \\ \dot{f}f' & 1 + f'^2 & \alpha' g'/2 & 0 \\ -\alpha' \dot{g}/2 & -\alpha' g'/2 & r^2 + (1-g)^2 f^2 & 0 \\ 0 & 0 & 0 & \delta_{\hat{\mu}\hat{\nu}} \end{bmatrix}, \\ [M_{MN}^{(2)}] &= [M_{MN}^{(1)}]^T. \end{aligned} \quad (18)$$

We also have $\det(M^{(1)}) = \det(M^{(2)})$ since $M_{MN}^{(1)} = M_{NM}^{(2)}$ and so we omit the brane index (i) on $\det(M^{(i)})$ in subsequent calculations.

The action for this ansatz simplifies to:

$$S = -2 \int V(f) \left((1 + \partial_m f \partial^m f) [r^2 + f^2(1 - g)^2] \right. \\ \left. + \frac{\alpha'^2}{4} \partial_m g \partial^m g - \frac{\alpha'^2}{4} (\epsilon^{mn} \partial_m f \partial_n g)^2 \right)^{1/2} d^{p+1}x \quad (19)$$

where $x^m = (x^0, x^1) = (t, r)$ and $g_{mn} dx^m dx^n = -dt^2 + dr^2$. For notational convenience we define the scalar quantity

$$\Sigma(t, r) = \frac{V(f)}{\sqrt{-\det(M)}} \quad (20)$$

in analogy with (6). The equation of motion for the tachyon is

$$\partial_m \left[\Sigma [r^2 + (1 - g)^2 f^2] \partial^m f - \frac{\alpha'^2 \Sigma}{4} (\epsilon^{ab} \partial_a f \partial_b g) \epsilon^{mn} \partial_n g \right] \\ = \Sigma (1 + \partial^m f \partial_m f) (1 - g)^2 f + \frac{V'(f) V(f)}{\Sigma}, \quad (21)$$

and the nontrivial component of the equation of motion for the gauge field is

$$\frac{\alpha'^2}{4} \partial_m [\Sigma \partial^m g - \Sigma (\epsilon^{ab} \partial_a f \partial_b g) \epsilon^{mn} \partial_n f] \\ = \Sigma (1 + \partial^m f \partial_m f) f^2 (g - 1). \quad (22)$$

Although these equations are somewhat cumbersome, inspection of (22) tells us that there should exist a solution $g(t, r)$ such that at $g = 1$, the vacuum, we have $\partial_m g = 0$. This is the asymptotic behavior which corresponds to a vortex solution; it is already known from [11] that the static solution $g(r)$ is a monotonically increasing function which varies between 0 and 1. Thus we shall only consider solutions with the asymptotic behavior $\partial_m g(t, r) \rightarrow 0$ as $g(t, r) \rightarrow 1$. We will take initial data $g(0, r) = g_i(r)$ such that $g_i(0) = 0$, $0 \leq g_i(r) \leq 1$ for all r and $g'_i(r) \geq 0$ for all r . In addition we will focus on initial tachyon profiles $f(0, r) = f_i(r)$ such that $f_i(0) = 0$ and $f'_i(r) > 0$ for all r . For these initial conditions the tachyon must start rolling for $r \neq 0$ due to its displacement from the unstable vacuum $V'(f) = 0$. Since the asymptotic $g_i(r \rightarrow \infty) \rightarrow 1$ is an exact solution of (22) and $g(t, r = 0) = 0$ by construction, we therefore expect $g(r, t)$ to increase towards unity for finite $r \neq 0$.

A. Solutions Near the Core of the Defect

As in [10], to analytically study the dynamics near the core of the vortex, $r = 0$, we make the ansatz:

$$f(t, r) \cong p(t)r, \quad g(t, r) \cong q(t)r \quad (23)$$

for small r . Dropping terms which are subleading in r yields a set of coupled ODEs for $p(t)$ and $q(t)$

$$p q^3 \ddot{q} - \ddot{p} q^4 + 2 \dot{p} q^3 \dot{q} - 2 p q^2 \dot{q}^2 + \frac{4}{\alpha'^2} p^3 q^2 + \frac{4}{\alpha'^2} p q^2 + \frac{2}{a^2} p q^4 = 0 \quad (24)$$

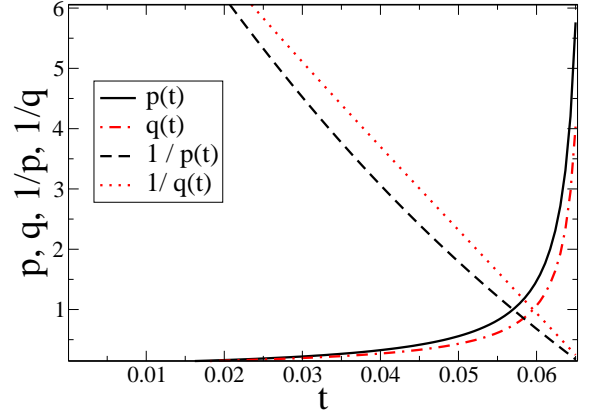


FIG. 1: Numerical solution for $p(t)$ and $q(t)$ of (24-25) showing the finite-time slope divergence. $1/p$ and $1/q$ are also shown, demonstrating the linearity of these functions near the critical time.

from (21) and

$$- p \ddot{p} q^3 + p^2 q^2 \ddot{q} + q^2 \ddot{q} - 2 q \dot{q}^2 + 2 p \dot{p} q^2 \dot{q} - 2 p^2 q \dot{q}^2 + \frac{4}{\alpha'^2} p^4 q \\ + \frac{8}{\alpha'^2} p^2 q + \frac{4}{\alpha'^2} q + \frac{2}{a^2} p^2 q^3 = 0 \quad (25)$$

from (22). Although (24,25) are difficult to solve analytically it is straightforward to verify that in the regime where \dot{p} , \dot{q} and higher derivatives are large compared to p , q there exists an approximate solution to (24-25) where both $p(t)$ and $q(t)$ are divergent in finite time t_c :

$$p(t) = \frac{p_0}{t_c - t}, \quad q(t) = \frac{q_0}{t_c - t} \quad (26)$$

in analogy with the kink solution of [10]. Numerical solutions to (24-25) agree with this prediction, as shown in figure 1.

B. Solutions Away From the Core of the Defect

We are interested in solutions where $\partial_m g \rightarrow 0$ as $g \rightarrow 1$ and where $f(t, r) \rightarrow \infty$ as $t \rightarrow \infty$. Since, as we have seen above, $g'(t, r = 0)$ is diverging in finite time, therefore g must be increasing to unity for $r \neq 0$ so that at late times we expect $g(t, r)$ to resemble a step function. Thus to study the dynamics away from the core of the defect we begin with the ansatz:

$$g(t, r) = 1 - \varepsilon \sigma(t, r) \quad (27)$$

and work only to leading order in ε . The leading order contribution to (21) decouples completely from $\sigma(t, r)$

$$\left[\ddot{f}(1 + f'^2) - f''(1 - \dot{f}^2) - 2 \dot{f} f' \dot{f}' - \frac{2f}{a^2} (1 - \dot{f}^2 + f'^2) \right] r \\ - \left[1 - \dot{f}^2 + f'^2 \right] f' = 0. \quad (28)$$

We can consistently find solutions of (28) by taking f to be a solution of the eikonal equation $1 - \dot{f}^2 + f'^2 = 0$. Subject to this constraint the second term in the square braces in (28) vanishes trivially. The constraint that the first term in the square braces in (28) vanishes is exactly the same as the equation of motion one would derive from $\mathcal{L} = -V(f)\sqrt{1 + \partial_m f \partial^m f}$, the Born-Infeld Lagrangian. The eikonal equation yields the Born-Infeld equation as a differential consequence, which is not surprising since this amounts to minimizing the action by setting $\mathcal{L} = 0$. Thus the PDE (28) is automatically satisfied when f is a solution of the eikonal equation. We find, then, that when $g(t, r) \cong 1$ the tachyon field must obey

$$\partial_m f \partial^m f + 1 = 0 \quad (29)$$

as in section II.

The ansatz (27) yields no simplification of (22); however, we may solve for $\sigma(t, r)$ given (29) based on more fundamental constraints. The argument of the square root in (19) must be nonnegative to ensure reality of the Lagrangian. Thus for $f(t, r)$ given by (29) the requirement that the Lagrangian be real translates to

$$\partial_m \sigma \partial^m \sigma - (\epsilon^{mn} \partial_m f \partial_n \sigma)^2 \geq 0.$$

or, with f given by (29),

$$-\left(\dot{\sigma}\sqrt{1 + f'^2} - \sigma' f'\right)^2 \geq 0. \quad (30)$$

For real fields it is clear that (30) can only be solved when the equality is taken. It is worth noting that since the Lagrangian vanishes when the equality is taken in (30), this constraint ensures that the full equations of motion are satisfied.

As in section II we can avoid the difficulties of caustic formation in the general solutions of (29) found in [17] by taking the Cauchy data to be linear and using the one-parameter family of solutions

$$f(t, r) = \alpha t + \sqrt{\alpha^2 - 1} r. \quad (31)$$

Reality of the Lagrangian requires

$$\alpha \dot{\sigma} - \sqrt{\alpha^2 - 1} \sigma' = 0.$$

This PDE is separable and we find the solution

$$g(t, r) = 1 - \varepsilon \exp\left(-\frac{r}{R}\right) \exp\left(-\frac{t}{R} \sqrt{\frac{\alpha^2 - 1}{\alpha^2}}\right) \quad (32)$$

where R is a separation constant. Note that the solution (32) becomes static in the limit $\alpha^2 \rightarrow 1$, the homogeneous rolling tachyon. In fact, for $\alpha^2 = 1$ any function $\sigma(t, r) = \sigma(r)$ satisfying the necessary boundary condition $\sigma(r \rightarrow \infty) \rightarrow 0$ will generate a solution. We will ultimately be interested in this limit.⁴

It is noteworthy that taking $g(t, r)$ close to unity (27) ultimately translates into the requirement that $\det(M)$ must vanish. The solutions (31-32) should be thought of as late-time asymptotics where $V(f) \rightarrow 0$ since $f \rightarrow \infty$. In this limit $\Sigma(t, r)$ defined in (20) has the indeterminant form $\frac{0}{0}$ as in section II. The quantity Σ is of some interest for two reasons. First, it parametrizes the manner in which we take the limits $V(f) \rightarrow 0$ and $\det(A) \rightarrow 0$ as we approach the vacuum state. Second, the form of Σ for $r \neq 0$ will determine the form of the energy-momentum tensor in that regime, as in the case of the kink. Following the discussion in section II we will use energy-momentum conservation to place constraints on the asymptotic form of Σ .

V. STRESS-ENERGY TENSOR

In this section we demonstrate that the vortex solutions found above give rise to the formation of a singularity in the stress-energy tensor of the tachyon field, which corresponds exactly with that of a codimension-two D-brane in the final state, whose tension has the value expected from string theory. We also derive the bulk stress-energy tensor for the leftover tachyon matter, which continues to roll even after the formation of the D-brane.

The stress-energy tensor for the action (14) is

$$T^{MN} = -\frac{V(T, Y_{(1)}^I - Y_{(2)}^I)}{r} \left[\sqrt{-\det M^{(1)}} (M_{(1)}^{-1})_S^{MN} + \sqrt{-\det M^{(2)}} (M_{(2)}^{-1})_S^{MN} \right] \quad (33)$$

where the subscript S denotes the symmetric part of the matrix, *i.e.*, $(M_{(i)}^{-1})_S^{MN} = \frac{1}{2}[(M_{(i)}^{-1})^{MN} + (M_{(i)}^{-1})^{NM}]$. The components of T^{MN} parallel to the vortex simplify to

$$T^{\hat{t}\hat{v}} = -\frac{2}{r} V(f) \sqrt{-\det(M)} \delta^{\hat{t}\hat{v}}. \quad (34)$$

For the components involving t, r, θ , it is useful to rewrite $T^{\mu\nu}$ in terms of Σ and the symmetrized cofactor matrix of $M_{\mu\nu}^{(i)}$, which we define as $C_{(i)}^{\mu\nu}$.⁵ Since $C_{(1)}^{\mu\nu} = C_{(2)}^{\mu\nu}$ we drop the brane index on the cofactor matrices. In the t, r, θ directions, then, we have

$$T^{\mu\nu} = \frac{2\Sigma}{r} C^{\mu\nu}. \quad (35)$$

The nonzero components of the cofactor matrix for the Lagrangian (19) are

$$C^{tt} = [r^2 + f^2(1 - g)^2] (1 + f'^2) + \frac{\alpha'^2}{4} g'^2,$$

⁴ The exact functional form of σ at large r and late times turns out to be of little importance to the ensuing analysis.

⁵ That is to say $C_{(i)}^{\mu\nu} = \det(M) (M_{(i)}^{-1})_S^{\mu\nu}$.

$$\begin{aligned}
C^{tr} &= -[r^2 + f^2(1-g)^2] \dot{f} f' - \frac{\alpha'^2}{4} \dot{g} g', \\
C^{rr} &= -[r^2 + f^2(1-g)^2] (1 - \dot{f}^2) + \frac{\alpha'^2}{4} \dot{g}^2, \\
C^{\theta\theta} &= -(1 - \dot{f}^2 + f'^2).
\end{aligned}$$

A. Normalization of the Potential

In [11] Sen finds that the action (14) provides a good effective description of the tachyon on the brane-antibrane system provided the potential $V(T)$ is chosen to satisfy the normalization constraint

$$\tau_{p-2} = 4\pi \int_0^\infty V(z) \sqrt{z^2(1 - \hat{G}(z))^2 + \frac{\alpha'^2}{4} (\hat{G}'(z))^2} dz \quad (36)$$

where $\tau_{p-2} = (2\pi)^2 \alpha' \tau_p$ is the tension of a $(p-2)$ -brane, $\hat{G}(z) = G(F^{-1}(z))$, and $\{f(r) = F(br), g(r) = G(br)\}$ are the static soliton solutions to be understood in the limit that $b \rightarrow \infty$. This constraint is necessary to ensure that the vortex solution has the correct tension to be interpreted as a $D(p-2)$ -brane. In the time-dependent case there is some ambiguity as to how to interpret (36) since this statement appears to depend on the functional form of the solutions, which would make the right-hand-side apparently time-dependent.⁶

But physically, it makes sense to impose (36) at $t \geq t_c$ since t_c is the time by which the brane has actually formed, and in the limit $t \rightarrow t_c$ the time-dependent solutions should coincide with the soliton solutions in the neighbourhood of $r = 0$. In the time-dependent case, it is $(t_c - t)^{-1}$ which tends to infinity (as $t \rightarrow t_c$) and plays the role of b . Although the exact functional forms of $G(z)$, $F(z)$ are not known for all z , we can infer from (26) that $G(z) \cong q_0 z$ and $F(z) \cong p_0 z$ near $z = 0$. Furthermore, we know that $G(z) \rightarrow 1$ for sufficiently large z by construction. We also have $F(z = 0) = 0$ and $F(z \neq 0) \neq 0$ so that $F^{-1}(0) = 0$ and $F^{-1}(z \neq 0) \neq 0$.

Let us consider the two terms under the square root in (36). The first term, $z^2(1 - \hat{G}(z))^2$, is small near $z = 0$ due to the overall multiplicative factor of z^2 . On the other hand, at large z this term is also small since $F^{-1}(z)$ is large and hence $\hat{G}(z) = G(F^{-1}(z)) \cong 1$. We conclude that the derivative term under the root in (36) dominates. At small z we have $F^{-1}(z) \cong p_0^{-1} z$ and thus $\hat{G}(z) \cong q_0 F^{-1}(z) \cong \frac{q_0}{p_0} z$ so that $\hat{G}'(z) \cong \frac{q_0}{p_0}$. For simplicity we take $\hat{G}'(z) \cong \frac{q_0}{p_0}$ for all z since this expression is multiplied by $V(z)$ which tends to zero quickly for large z . We find then that

$$\frac{q_0}{p_0} \int_{-\infty}^\infty e^{-z^2/a^2} dz = 4\pi. \quad (37)$$

The normalization (37) is equivalent to $a = 4\sqrt{\pi} \frac{p_0}{q_0}$. We shall see later on that the relation $a = 4\sqrt{\pi} \frac{p_0}{q_0}$ may be equivalently viewed as a constraint on the arbitrary function $\Sigma(t, r)$.

B. Stress-Energy Tensor at $r = 0$

At $r \cong 0$ and $t \rightarrow t_c$ the solutions (26) are valid and the Hamiltonian is

$$\begin{aligned}
T^{00} &= \frac{2\Sigma}{r} \left([r^2 + f^2(1-g)^2] (1 + f'^2) + \frac{\alpha'^2}{4} g'^2 \right) \\
&\cong \frac{\tau_p}{r} \frac{q_0 \alpha'}{t_c - t} \exp \left(-\frac{p_0^2 r^2}{a^2 (t_c - t)^2} \right)
\end{aligned}$$

to leading order in r . Then, using $a = 4\sqrt{\pi} \frac{p_0}{q_0}$,

$$\begin{aligned}
\lim_{t \rightarrow t_c} T^{00} &= 4\pi \tau_{p-2} \frac{\delta(r)}{r} \\
&= \tau_{p-2} \delta(r \cos \theta) \delta(r \sin \theta)
\end{aligned}$$

and the components parallel to the vortex are

$$\begin{aligned}
T^{\hat{\mu}\hat{\nu}} &= -\frac{2}{r} \delta^{\hat{\mu}\hat{\nu}} V(f) \sqrt{-\det(M)} \\
&\rightarrow -\tau_{p-2} \delta^{\hat{\mu}\hat{\nu}} \delta(r \cos \theta) \delta(r \sin \theta).
\end{aligned}$$

The remaining components, T^{11} and T^{01} , are vanishing at $r = 0$. The angular component $T^{22} = T^{\theta\theta}$ contains a delta function at $r = 0$; however this is an artifact of θ being a bad coordinate at $r = 0$; it can be seen that $T_{\theta\theta} = 0$, and going to Cartesian coordinates confirms that the $T_{\mu\nu} = 0$ for the transverse coordinates, as should be the case for a D-brane. This result is the same as in the static case [11].

C. Stress-Energy Tensor at $r > 0$

For $r > 0$ at late times the solutions (31,32) are valid. For simplicity we take $\alpha^2 = 1$ and work only to leading order in ε . The Hamiltonian is

$$T^{00} \cong 2r \Sigma(t, r)$$

and the remaining components of T^{MN} vanish at late times for $r > 0$. Notice that conservation of energy $\partial_M T^{MN} = 0$ at large r forces

$$\Sigma(t, r) = \Sigma(r).$$

That is, Σ is an arbitrary function of r as in section II.

To summarize, we find that in the limit of condensation the energy momentum tensor is identical to that of a $D(p-2)$ -brane

$$\begin{aligned}
T^{00} &= \tau_{p-2} \delta(r \cos \theta) \delta(r \sin \theta) + 2r \Sigma(r) \\
T^{11} &= T^{22} = 0 \\
T^{\hat{\mu}\hat{\nu}} &= -\delta^{\hat{\mu}\hat{\nu}} \tau_{p-2} \delta(r \cos \theta) \delta(r \sin \theta)
\end{aligned} \quad (38)$$

⁶ Arguments are presented in [11] for why (36) is in fact only a constraint on $V(T)$ and not on the solutions $T(x)$ themselves.

with all off-diagonal components vanishing. The extra bulk energy density $2r\Sigma(r)$ is similar to the result in [19] and section II and corresponds to tachyon matter rolling toward $T \rightarrow \infty$ in the bulk.

D. Conservation of Energy

We can constrain $\Sigma(r)$ using conservation of energy. Initially the system consists of two Dp -branes with energy density $2\tau_p V_2$ in the $(p-2)$ -dimensional subspace spanned by $\{x^{\hat{\mu}}\}$, where V_2 is the volume of the 2-dimensional subspace spanned by $\{r, \theta\}$. At late times, after the codimension 2 brane and its antibrane have formed, the energy density in the $(p-2)$ -dimensional space is given by the sum of the $D(p-2)$ -brane tensions, $2\tau_{p-2}$, and the energy density due to tachyon matter $4\pi \int r^2 \Sigma(r) dr$. Conservation of energy thus implies

$$\begin{aligned} 2\tau_p V_2 &= 2\tau_{p-2} + 4\pi \int dr r^2 \Sigma(r) \\ (2V_2 - 2(2\pi)^2 \alpha') \tau_p &= 4\pi \int dr r^2 \Sigma(r). \end{aligned}$$

Since $\Sigma(r)$ is arbitrary we can take

$$\Sigma(r) = \frac{\tau_p}{r} + \frac{\tilde{\Sigma}(r)}{4\pi r^2} \quad (39)$$

where $\tilde{\Sigma}(r)$ satisfies the constraint

$$\int dr \tilde{\Sigma}(r) = -8\pi^2 \alpha' \tau_p. \quad (40)$$

The conditions (39),(40) are equivalent to (37) and may be thought of as an alternative to the normalization (36).

VI. INCLUSION OF MASSLESS GAUGE FIELDS

We will now restrict ourselves to a $(5+1)$ -dimensional spacetime with $\{M, N\} = \{0, 1, \dots, 5\}$, $\{\mu, \nu\} = \{0, 1, 2\}$ and $\{\hat{\mu}, \hat{\nu}\} = \{3, 4, 5\}$. There are two gauge fields in the problem: $A_{(1)}^M$ and $A_{(2)}^M$, or equivalently $A_+^M = A_{(1)}^M + A_{(2)}^M$ and $A_-^M = A_{(1)}^M - A_{(2)}^M$, which have different couplings to the tachyon. We have already shown that A_-^μ is the field which condenses in the vortex, hence its associated gauge symmetry is spontaneously broken. For reheating it is thus $A_+^{\hat{\mu}}$ which most closely resembles the Standard Model photon. We will ignore fluctuations of the heavy fields $A_-^{\hat{\mu}}$, A_+^μ , and A_-^μ , keeping only the background solution for A_-^μ (which was given in section IV), and the fluctuations of the photon $A_+^{\hat{\mu}}$. This leads to considerable simplification since it ensures that $D_{\hat{\mu}} T = 0$. To compute the production of photons in the time-dependent background, we want to expand the action (14) to quadratic order in $A_+^{\hat{\mu}}$.

The matrix $M_{MN}^{(i)}$ of eq. (15) can be written in block diagonal form as

$$[M_{MN}^{(i)}] = \begin{bmatrix} [\mathcal{V}_{\mu\nu}^{(i)}] & [S_{\mu\hat{\nu}}^{(i)}] \\ -[S_{\mu\hat{\nu}}^{(i)}]^T & [\delta_{\hat{\mu}\hat{\nu}} + \alpha' F_{\hat{\mu}\hat{\nu}}^{(i)}] \end{bmatrix} \quad (41)$$

where $\mathcal{V}_{\mu\nu}^{(i)} = M_{\mu\nu}^{(i)}$ is the contribution from the vortex background given in (18), $S_{\mu\hat{\nu}}^{(i)} = \alpha' \partial_\mu A_{\hat{\nu}}^{(i)}$ is the contribution from $\{t, r, \theta\}$ derivatives of $A_{\hat{\mu}}^{(i)}$, and $F_{\hat{\mu}\hat{\nu}}^{(i)}$ is the field strength tensor for $A_{\hat{\mu}}^{(i)}$. Using a well-known identity for determinants we can write

$$\det(M^{(i)}) = \det \mathcal{V}^{(i)} \det \left(\mathbf{1} + \alpha' F^{(i)} + S_{(i)}^T \mathcal{V}_{(i)}^{-1} S_{(i)} \right). \quad (42)$$

Expanding $\det(\mathbf{1} + \alpha' F^{(i)} + S_{(i)}^T \mathcal{V}_{(i)}^{-1} S_{(i)})$ to quadratic order in $A_{\hat{\mu}}^{(i)}$, the action (14) becomes

$$\begin{aligned} S \cong & -\alpha'^2 \frac{1}{4} \int d^{p+1} x V(f) \sqrt{-\det \mathcal{V}} \left((\mathcal{V}^{-1})^{\mu\nu}_{(S)} \partial_\mu A_{\hat{\mu}}^+ \partial_\nu A_{\hat{\mu}}^+ \right. \\ & \left. + \partial_{\hat{\mu}} A_{\hat{\nu}}^+ \partial^{\hat{\mu}} A_{\hat{\nu}}^+ \right) \end{aligned} \quad (43)$$

where we have chosen the gauge $\partial_{\hat{\mu}} A_{\hat{\nu}}^+ = 0$ and disregarded the piece which does not depend on $A_{\hat{\nu}}^+$. We omit the brane index on $\mathcal{V}_{(i)}$ since the determinant and the symmetric part of $\mathcal{V}_{(i)}^{-1}$ are equal for both $i = \{1, 2\}$. Defining an effective metric G_{MN} by

$$[G_{MN}] = \begin{bmatrix} [\mathcal{V}_{\mu\nu}^{(S)}] & [0] \\ [0] & [\delta_{\hat{\mu}\hat{\nu}}] \end{bmatrix} \quad (44)$$

the action (43) may be written as

$$S = -\frac{\alpha'^2}{4} \int V(f) \sqrt{-G} G^{MN} \delta^{\hat{\mu}\hat{\nu}} \partial_M A_{\hat{\mu}}^+ \partial_N A_{\hat{\nu}}^+ d^{p+1} x. \quad (45)$$

From (45) one sees that the fluctuations of the photon behave like a collection of massless scalar fields propagating in a nonflat spacetime described by the metric G_{MN} , with a position- and time-dependent gauge coupling given by $g^2 = 1/V(f(t, r))$.

To get an intuitive sense for the behavior of the action (45) we note that the stress-energy tensor derived in section V can be written as

$$T^{MN} = -\frac{2}{r} V(f) \sqrt{-G} G^{MN}.$$

In the limit of condensation, T^{MN} is given by (38), so that once the brane has formed the action (45) reduces to a description of gauge fields propagating in a $(3+1)$ -dimensional Minkowski space, with an additional component which couples the gauge fields to the tachyon matter density in the bulk. In other words, the effective metric G_{MN} starts off being smooth throughout the bulk, but within the time t_c , its support collapses to become a delta function $\delta^{(2)}(\vec{x})$ in the relevant extra dimensions $\{r, \theta\}$.

The equations of motion resulting from the effective action (45) are difficult to solve analytically since the effective metric G_{MN} depends nontrivially on both r and t and is nondiagonal in the subspace of $\{t, r\}$. For this reason we would like to propose a simplified model of the condensation which captures the essential features of the action (45). We have derived solutions for the vortex background valid at small r , $r \lesssim (t_c - t)$, and at large r , $r \gtrsim (t_c - t)$. Similarly, the energy momentum tensor we have derived corresponding to these solutions has very different behavior in the $r \leq (t_c - t)$ and $r > (t_c - t)$ regions of the spacetime.

For $r \leq (t_c - t)$ the energy momentum tensor contracts to a delta function centered at $r = 0$, with $(t_c - t)$ playing the role of the small parameter which regularizes the delta function. That is to say,

$$rT^{MN} \cong \frac{1}{t_c - t} \exp\left(-\frac{p_0^2 r^2}{a^2(t_c - t)^2}\right) H^{MN}$$

at small r , where the matrix entries H^{00} , $H^{\hat{\mu}\hat{\nu}}$ are finite as $t \rightarrow t_c$ and the remaining components of H^{MN} tend to zero (near $r = 0$) as $t \rightarrow t_c$.

In the $r > (t_c - t)$ region, the energy momentum tensor has quite different behavior. After condensation of the defect has completed at $t = t_c$, the energy density in the bulk ($r > 0$) is due entirely to tachyon matter, while the part of the stress-energy which is going into the tension of the defect vanishes in this region. Hence

$$rT^{MN} \rightarrow 2\delta_0^M \delta_0^N r^2 \Sigma(r)$$

as $t \rightarrow t_c$, for $r > (t_c - t)$.

In our simplified model of the particle production due to the tachyon condensation we therefore split the energy momentum tensor into brane and bulk pieces $T^{MN} = T_{\text{brane}}^{MN} + T_{\text{bulk}}^{MN}$ where T_{brane}^{MN} contracts to a delta function as $t \rightarrow t_c$ and $T_{\text{bulk}}^{MN} \rightarrow 2\delta_0^M \delta_0^N r^2 \Sigma(r)$ in the same limit. The action (45) then splits into two components $S = S_{\text{bulk}} + S_{\text{brane}}$. We expect most of the particle production to occur near the end of the condensation, when the background tachyon field is becoming singular near the vortex, so the best approximations for the simplified gauge field action are those which describe the exact expression most accurately near $t = t_c$:

$$S_{\text{brane}} \propto - \int \frac{1}{t_c - t} \exp\left(-\frac{p_0^2 r^2}{a^2(t_c - t)^2}\right) H^{MN} \delta^{\hat{\mu}\hat{\nu}} \partial_M A_{\hat{\mu}}^+ \partial_N A_{\hat{\nu}}^+ dt dr d\theta dx^{\hat{\alpha}}$$

and

$$S_{\text{bulk}} \propto - \int r^2 \Sigma(r) \left(-\delta^{\hat{\mu}\hat{\nu}} \dot{A}_{\hat{\mu}}^+ \dot{A}_{\hat{\nu}}^+\right) dt dr d\theta dx^{\hat{\alpha}}.$$

At earlier times the coefficient of the bulk part of the Lagrangian would have time dependence, and the bulk Lagrangian would contain contributions from all the derivative of the gauge field, but this form is valid close to t_c .

Finally we argue that the bulk part of the action can be ignored. To this end, let us change to coordinates which are comoving with the contraction of the vortex core:

$$\tilde{r} = \frac{r}{t_c - t}, \quad \tilde{t} = t_c - t. \quad (46)$$

In terms of these coordinates the “small r ” solutions are valid for $\tilde{r} \leq 1$ and the “large r ” solutions are valid for $\tilde{r} > 1$. The Jacobian of this transformation is $-\tilde{t}$ so that

$$S_{\text{brane}} \propto - \int \exp\left(-\frac{p_0^2 \tilde{r}^2}{a^2}\right) H^{MN} \delta^{\hat{\mu}\hat{\nu}} \partial_M A_{\hat{\mu}}^+ \partial_N A_{\hat{\nu}}^+ d\tilde{t} d\tilde{r} d\theta dx^{\hat{\alpha}}.$$

To lowest order in \tilde{r}^2 the matrix entries H^{MN} in terms of these new coordinates are all constant. Consider now the piece of the action which couples to the tachyon matter density in the bulk, written in terms of these new coordinates:

$$S_{\text{bulk}} \propto - \int \tilde{r}^2 \tilde{t}^2 \Sigma(\tilde{r}\tilde{t}) \left(-\delta^{\hat{\mu}\hat{\nu}} \dot{A}_{\hat{\mu}}^+ \dot{A}_{\hat{\nu}}^+\right) d\tilde{t} d\tilde{r} d\theta dx^{\hat{\alpha}}.$$

Since $\Sigma(z)$ is not singular at $z = 0$, the bulk piece of the action become negligible near the end of the contraction $\tilde{t} \rightarrow 0$. This is a consequence of the fact that as the condensation proceeds the gauge field is confined to the descendant brane.

VII. SIMPLIFIED MODEL OF REHEATING

In the previous section we argued that the gauge field couples most strongly to the part of the tachyon background which is collapsing to form the defect. This closely resembles a gauge theory defined on a manifold in which a two-dimensional subspace which is shrinking with time. As a simplified model of the interaction we thus consider a massless spin-1 field

$$S = -\frac{1}{4} \int \sqrt{-g} g^{MA} g^{NB} F_{MN} F_{AB} d^{p+1}x$$

propagating in a FRW-like background

$$g_{MN} dx^M dx^N = -dt^2 + R(t)^2 (d\tilde{r}^2 + \tilde{r}^2 d\theta^2) + \delta_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}}. \quad (47)$$

The coordinate \tilde{r} in (47) is fixed with the expansion and is thus corresponds to \tilde{r} defined in (46). However for simplicity of notation we will drop the tilde and write r instead of \tilde{r} in the remainder of the paper. We take r to be dimensionless while t , $x^{\hat{\mu}}$ and R have dimensions of length.

If we restrict ourselves to configurations with $A^\mu = 0$, $A^{\hat{\mu}} \neq 0$ ⁷ and impose the gauge condition $\partial_{\hat{\mu}} A^{\hat{\mu}} = 0$ then

⁷ This restriction will only underestimate the reheating. Since we want to show that the reheating can be efficient, this approximation will not weaken the ensuing argument.

the action simplifies to

$$S = -\frac{1}{2} \int \sqrt{-g} g^{MN} \delta^{\hat{\mu}\hat{\nu}} \partial_M A_{\hat{\mu}} \partial_N A_{\hat{\nu}} d^{p+1}x \quad (48)$$

Notice that for the metric (47) and the gauge field configuration $A^\mu = 0$ we have chosen, one has $\nabla_M A_{\hat{\mu}} = \partial_M A_{\hat{\mu}}$ and $R_{MN} A^M A^N = R_{\hat{\mu}\hat{\nu}} A^{\hat{\mu}} A^{\hat{\nu}} = 0$.

We will impose homogeneous boundary conditions at $r = 1$ and take the scale factor in (47) to be

$$R(t) = \begin{cases} R_0 & \text{if } t < 0; \\ R_0 - \eta t & \text{if } 0 \leq t \leq t_c; \\ R_0 - \eta t_c = \epsilon & \text{if } t > t_c. \end{cases} \quad (49)$$

where R_0 represents the initial radial size of the extra dimensions. This approximation for the time-dependence of the vortex core is the simplest form which has the same qualitative behavior as the true background, while still allowing us to solve analytically for the gauge field wave functions in the background. A shortcoming of this approximation is that R is discontinuous at the interfaces, which leads to an ultraviolet divergence in the production of gauge bosons. The behavior of the actual $R(t)$ is smooth, and must yield a finite amount of particle production [5]. In addition, we will find a separate UV divergence in the particle production in the limit as the vortex core thickness goes to zero. This is presumably an artifact of the effective field theory which is not present in the full string theory, and we deal with it by introducing the cutoff ϵ on the final radius of the defect core, which we will take to be of order the string length l_s .

A. Gauge Field Solutions

The first step in computing the production of photons in the time-dependent background is to solve their equation of motion following from (48):

$$-2\frac{\dot{R}}{R}\dot{A}^{\hat{\rho}} - \ddot{A}^{\hat{\rho}} + \frac{A^{\hat{\rho}''}}{R^2} + \frac{A^{\hat{\rho}'}}{rR^2} + \frac{\partial_\theta^2 A^{\hat{\rho}}}{r^2 R^2} + \partial^{\hat{\mu}} \partial_{\hat{\mu}} A^{\hat{\rho}} = 0 \quad (50)$$

where the dot and prime denote differentiation with respect to t and r , respectively.

Equation (50) separates as

$$A^{\hat{\rho}}(t, r, \theta, x^{\hat{\mu}}) = \phi(t) \varphi(r) \Theta(\theta) \chi^{\hat{\rho}}(x^{\hat{\mu}}) \quad (51)$$

where

$$\partial^{\hat{\mu}} \partial_{\hat{\mu}} \chi^{\hat{\rho}} = -k^2 \chi^{\hat{\rho}}, \quad \partial_{\hat{\nu}} \chi^{\hat{\rho}} = 0, \quad k^2 = k^{\hat{\mu}} k_{\hat{\mu}} = \vec{k} \cdot \vec{k} \quad (52)$$

$$\partial_\theta^2 \Theta = -m^2 \Theta \quad (53)$$

$$\varphi'' + \frac{1}{r} \varphi' + \left(c^2 - \frac{m^2}{r^2} \right) \varphi = 0 \quad (54)$$

$$\ddot{\phi} + 2\frac{\dot{R}}{R}\dot{\phi} + \left(\frac{c^2}{R^2} + k^2 \right) \phi = 0 \quad (55)$$

and c is a separation constant.

The particular solutions of (52) are labeled by the momenta \vec{k} in the 3 large dimensions ($\{x^4, x^5, x^6\}$) and we take them to be normalized according to

$$\int \chi_{\vec{k}}^{\hat{\mu}} \chi_{\vec{k}'}^{\hat{\mu}} dx^4 dx^5 dx^6 = \delta_{\vec{k}\vec{k}'}.$$

The particular solution of (53) is a sum of sines and cosines. The odd parity and even parity modes (under $\theta \rightarrow -\theta$) do not mix with the even ones, and for simplicity, we restrict our attention to the even modes, which include the massless one. This can underestimate the efficiency of the reheating by a factor of 2 at most. The solution of (53) is thus

$$\Theta_m(\theta) = \frac{1}{\sqrt{\pi}} \cos(m\theta).$$

These are orthogonal for different values of m , and requiring that the solution be single-valued restricts m to be an integer.

So far we have not been specific about the geometry of the two extra dimensions around which the original annihilating branes were wrapped. One simple possibility is a 2-sphere, where the descendant brane and antibrane (vortices) form at antipodal points. Since all the singular behavior of the tachyon background is localized near these points, the curvature and topology in the bulk should have little effect on particle production near the defects. To simplify the mathematics, we therefore replace either of the two hemispheres of the sphere with a flat disk, in the coordinate region $r \leq 1$. The correct boundary condition on radial eigenfunctions of the bulk Laplacian is that their derivatives vanish at $r = 1$, so that they are smooth at the interface where the two halves of the space are glued together.

The solution of (54), subject to the boundary condition $\varphi'(r = 1) = 0$ and the requirement that $\varphi(r)$ be regular at the origin, is

$$\varphi_{mn}(r) = \frac{\sqrt{2}}{J_{m+1}(c_{mn})} J_m(c_{mn}r) \quad (56)$$

where c_{mn} is the n th zero of $J'_m(r)$. The solutions (56) are orthogonal for different values of n . The zero mode $n = 1$, $m = 0$ must be treated separately; it is the constant solution, where $c_{01} = 0$ and $\varphi_{01}(r) = 1$.

The solution of (55) depends on the scale factor. For $t < 0$ and $t > t_c$ the solutions are trivial and are given by

$$\phi_{mn}(t) = \frac{1}{R_0 \sqrt{2\omega_{mn}}} (a_{mn} e^{-i\omega_{mn}t} + a_{mn}^\dagger e^{i\omega_{mn}t}) \quad (57)$$

and

$$\phi_{mn}(t) = \frac{1}{\epsilon \sqrt{2\bar{\omega}_{mn}}} (d_{mn} e^{-i\bar{\omega}_{mn}t} + d_{mn}^\dagger e^{i\bar{\omega}_{mn}t}) \quad (58)$$

respectively. We have defined $\omega_{mn}^2 = \frac{c_{mn}^2}{R_0^2} + k^2$ and $\bar{\omega}_{mn}^2 = \frac{c_{mn}^2}{\epsilon^2} + k^2$. The multiplicative factors in (57) and

(58) are introduced for later convenience and ensure that a_{mn} and d_{mn} will be properly normalized annihilation operators in the appropriate spacetime region when the gauge field is quantized. It will be convenient for what follows to introduce phase-shifted annihilation and creation operators in the region $t > t_c$: $\bar{d}_{mn} = e^{-i\bar{\omega}_{mn}t_c} d_{mn}$ and $\bar{d}_{mn}^\dagger = e^{i\bar{\omega}_{mn}t_c} d_{mn}^\dagger$. In terms of these operators (58) becomes

$$\phi_{mn}(t) = \frac{1}{\epsilon\sqrt{2\bar{\omega}_{mn}}} \left(\bar{d}_{mn} e^{-i\bar{\omega}_{mn}(t-t_c)} + \bar{d}_{mn}^\dagger e^{i\bar{\omega}_{mn}(t-t_c)} \right).$$

We are suppressing the dependence of the annihilation/creation operators on the 3-momenta \vec{k} .

In the region $0 < t < t_c$ where the scale factor depends nontrivially on time, the solution of (55) is

$$\phi_{mn}(t) = \frac{1}{\sqrt{R_0 - \eta t}} \left(B_{mn} J_{p_{mn}} \left[\frac{k}{\eta} (R_0 - \eta t) \right] + C_{mn} J_{-p_{mn}} \left[\frac{k}{\eta} (R_0 - \eta t) \right] \right) \quad (59)$$

where $p_{mn} = \frac{1}{2\eta} \sqrt{\eta^2 - 4c_{mn}^2}$. Some comments are in order concerning this solution, which has different behavior for massive and massless modes.

In the massless case $c_{mn} = 0$, $p_{mn} = 1/2$ and J_p , J_{-p} are linearly independent, since p is noninteger. The constants B_{mn} and C_{mn} in (59) should be interpreted as independent, real-valued constants so that $\phi_{mn}(t)$ is real.

In the massive case $c_{mn} \neq 0$, $\eta^2 \leq 1$,⁸ and the order of the Bessel functions in (59), p_{mn} , is pure imaginary. Such Bessel functions are complex-valued and there are several options for constructing real solutions. Since $(J_\nu(x))^* = J_{\nu^*}(x)$ for real x , we can impose $C_{mn} = B_{mn}^\dagger$. Calculating the Wronskian of J_{is} , J_{-is} verifies that for real $s \neq 0$ these two solutions are linearly independent. Since we do not need to explicitly quantize the field in the region $0 < t < t_c$, the dagger can be thought of simply as complex conjugation. The interpretation of B_{mn} and B_{mn}^\dagger as annihilation and creation operators is unnecessary, since there are no asymptotic states in this region.

We summarize this subsection by putting these results together to write the general solution of (50) as

$$A^{\hat{\rho}}(x^M) = \sum_{m,n} \sum_{\vec{k}} \chi_{\vec{k}}^{\hat{\rho}}(x^{\hat{\mu}}) \Theta_m(\theta) \phi_{mn}(t) \varphi_{mn}(r).$$

where

$$\phi_{mn}(t) = \begin{cases} \frac{1}{R_0\sqrt{2\omega_{mn}}} (a_{mn} e^{-i\omega_{mn}t} + a_{mn}^\dagger e^{i\omega_{mn}t}) & \text{if } t < 0; \\ \frac{1}{\sqrt{R_0 - \eta t}} \left(B_{mn} J_{p_{mn}} \left[\frac{k}{\eta} (R_0 - \eta t) \right] + C_{mn} J_{-p_{mn}} \left[\frac{k}{\eta} (R_0 - \eta t) \right] \right) & \text{if } 0 \leq t \leq t_c; \\ \frac{1}{\epsilon\sqrt{2\bar{\omega}_{mn}}} (\bar{d}_{mn} e^{-i\bar{\omega}_{mn}(t-t_c)} + \bar{d}_{mn}^\dagger e^{i\bar{\omega}_{mn}(t-t_c)}) & \text{if } t > t_c. \end{cases}$$

B. Spectra of Produced Particles

The next step is to impose continuity of $A^{\hat{\mu}}$ and $\partial_M A^{\hat{\mu}}$ at $t = 0$ and $t = t_c$ in order to compute the Bogoliubov coefficients, which relate the annihilation and creation operators for $t > t_c$ (\bar{d}_{mn} and \bar{d}_{mn}^\dagger) to those in the region $t < 0$ (a_{mn} and a_{mn}^\dagger).⁹ Smoothness of the solutions at the interfaces is ensured by the continuity of ϕ_{mn} and $\dot{\phi}_{mn}$, since ϕ_{mn} is the only part of $A^{\hat{\rho}}$ which changes between the different spacetime regions.

⁸ One might expect $\eta^2 \leq 1$ on physical grounds. In fact, for the ensuing arguments to hold one need only restrict $\eta \leq 3.7$ to ensure that p_{mn} is pure imaginary and nonzero for all massive modes.

⁹ We are ultimately interested in calculating the number of d_{mn} quanta in the vacuum annihilated by a_{mn} . For this purpose it is just as good to use \bar{d}_{mn} , \bar{d}_{mn}^\dagger as d_{mn} , d_{mn}^\dagger since the phase shift cancels out of the number operator. $d_{mn}^\dagger d_{mn} = \bar{d}_{mn}^\dagger \bar{d}_{mn}$.

1. Massive Modes

For the massive modes continuity of $\phi_{mn}(t)$ and $\dot{\phi}_{mn}(t)$ at $t = 0$ implies

$$\begin{pmatrix} a_{mn} \\ a_{mn}^\dagger \end{pmatrix} = U \begin{pmatrix} B_{mn} \\ B_{mn}^\dagger \end{pmatrix} = \begin{bmatrix} u_1 & u_2 \\ u_2^\dagger & u_1^\dagger \end{bmatrix} \begin{pmatrix} B_{mn} \\ B_{mn}^\dagger \end{pmatrix}. \quad (60)$$

The entries of the matrix U are given by

$$\begin{aligned} u_1 &= \left(\sqrt{\frac{R_0\omega_{mn}}{2}} + \frac{i\eta}{2\sqrt{2}R_0\omega_{mn}} \right) J_{p_{mn}}(kR_0/\eta) \\ &\quad - ik\sqrt{\frac{R_0}{2\omega_{mn}}} J'_{p_{mn}}(kR_0/\eta) \\ u_2 &= \left(\sqrt{\frac{R_0\omega_{mn}}{2}} + \frac{i\eta}{2\sqrt{2}R_0\omega_{mn}} \right) J_{-p_{mn}}(kR_0/\eta) \\ &\quad - ik\sqrt{\frac{R_0}{2\omega_{mn}}} J'_{-p_{mn}}(kR_0/\eta). \end{aligned}$$

Continuity at $t = t_c$ similarly gives

$$\begin{pmatrix} \bar{d}_{mn} \\ \bar{d}_{mn}^\dagger \end{pmatrix} = V \begin{pmatrix} B_{mn} \\ B_{mn}^\dagger \end{pmatrix} = \begin{bmatrix} v_1 & v_2 \\ v_2^\dagger & v_1^\dagger \end{bmatrix} \begin{pmatrix} B_{mn} \\ B_{mn}^\dagger \end{pmatrix}. \quad (61)$$

The matrix entries v_i are obtained from u_i by replacing R_0 with ϵ and ω_{mn} with $\bar{\omega}_{mn}$.

From (60), (61) we can write

$$\begin{pmatrix} \bar{d}_{mn} \\ \bar{d}_{mn}^\dagger \end{pmatrix} = VU^{-1} \begin{pmatrix} a_{mn} \\ a_{mn}^\dagger \end{pmatrix} = \begin{bmatrix} \alpha_{mn} & \beta_{mn} \\ \beta_{mn}^* & \alpha_{mn}^* \end{bmatrix} \begin{pmatrix} a_{mn} \\ a_{mn}^\dagger \end{pmatrix}.$$

The last equality defines the Bogoliubov coefficients. Note that there is *no summation* implied over any of the indices in the above expression. The indices m and n label the modes of the in- and out-states, and the summation which appears in the general definition of the Bogoliubov coefficients is not present here.

We can now determine the spectrum of Kaluza-Klein (KK) excitations of the photon which is produced in the tachyon vortex background. Observers in the future see a spectrum of massive particles in the final state given by

$$N_{M>0}^{mn}(k) = |\beta_{mn}|^2 = \frac{1}{|\det U|^2} |u_1 v_2 - u_2 v_1|^2.$$

The determinant is

$$\begin{aligned} \det U &= \det V = -\frac{2i\eta}{\pi} \sin(\pi p_{mn}) \\ &= \frac{2\eta}{\pi} \sinh\left(\frac{\pi}{2\eta} \sqrt{4c_{mn}^2 - \eta^2}\right) \end{aligned} \quad (62)$$

which may be obtained by using the Wronskian of J_p and J_{-p} . The fact that the two determinants are equal ensures the appropriate normalization $|\alpha_{mn}|^2 - |\beta_{mn}|^2 = 1$ of the Bogoliubov coefficients. The explicit expression for $N_{M>0}^{mn}(k)$ can be obtained analytically, but it is complicated and we do not write it out here; instead we will give numerical results.

The mass of a KK mode with quantum numbers m, n is c_{mn}/ϵ , which increases with m and n . Figures 2-3 illustrate the dependence of $N_{M>0}^{mn}(k)$ on the 3-momentum k for the lightest few massive modes. The parameters of the vortex background are taken to be $R_0 = 10l_s$, $\epsilon = l_s$ and $\eta = 1$. (The dependence of $N_{M>0}^{mn}(k)$ on R_0 , ϵ and η is essentially the same as that of the spectrum for massless modes, $N_{M=0}(k)$, which we will discuss in the next section.)

2. Massless Modes

The analysis for the massless modes proceeds similarly to the calculations in the preceding section, but more simply since in this case $p_{mn} = 1/2$ and $\omega_{mn} = \bar{\omega}_{mn} = k$. The Bogoliubov coefficients do not depend on the mode indices:

$$\begin{pmatrix} \bar{d}_{mn} \\ \bar{d}_{mn}^\dagger \end{pmatrix} = \begin{bmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{bmatrix} \begin{pmatrix} a_{mn} \\ a_{mn}^\dagger \end{pmatrix}.$$

The observed spectrum of particles in the final state

$$\begin{aligned} N_{M=0}(k) &= |\beta|^2 \\ &\cong \frac{1}{\epsilon^2} \frac{\eta\pi R_0}{8k} \left(J_{\frac{1}{2}}(kR_0/\eta)^2 + J_{\frac{3}{2}}(kR_0/\eta)^2 \right) \end{aligned} \quad (63)$$

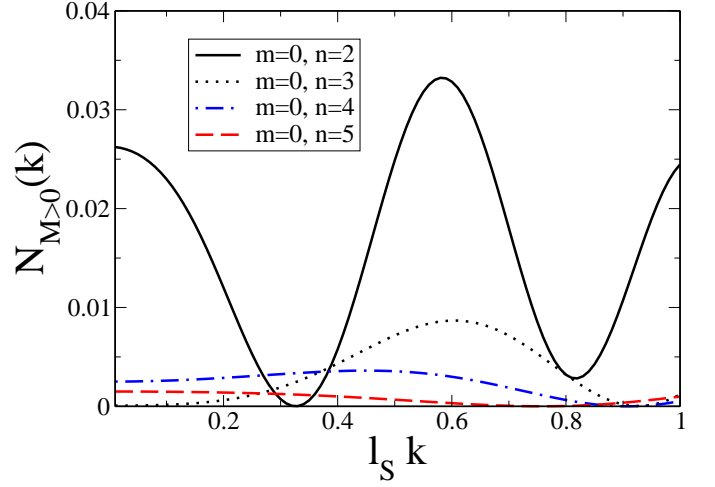


FIG. 2: $N_{M>0}^{mn}(k)$ versus k for increasing values of the radial quantum number n showing decreased production of heavier modes.

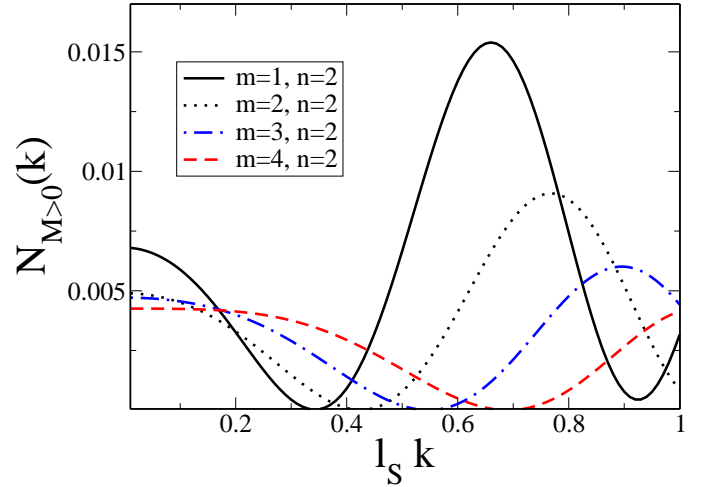


FIG. 3: Same as fig. 2, but varying the angular quantum number m .

where the second line shows the leading small- ϵ behavior. The exact result can also be obtained in closed form. Using explicit representations of $J_{n/2}$, we will be able to integrate this expression exactly to obtain the energy density of produced radiation. Figures 4-6 show plots of $N_{M=0}(k)$ as a function of k for various values of the parameters R_0 , ϵ and η .

C. Energy Density

To find the total energy density of radiation produced by the massive modes we should integrate over all k and sum over the mode indices m and n and the number of polarizations (3 for massive vector bosons):

$$\rho_{M>0} = 3 \sum_{m,n} \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + \frac{c_{mn}^2}{\epsilon^2}} N_{M=0}^{mn}(k). \quad (64)$$

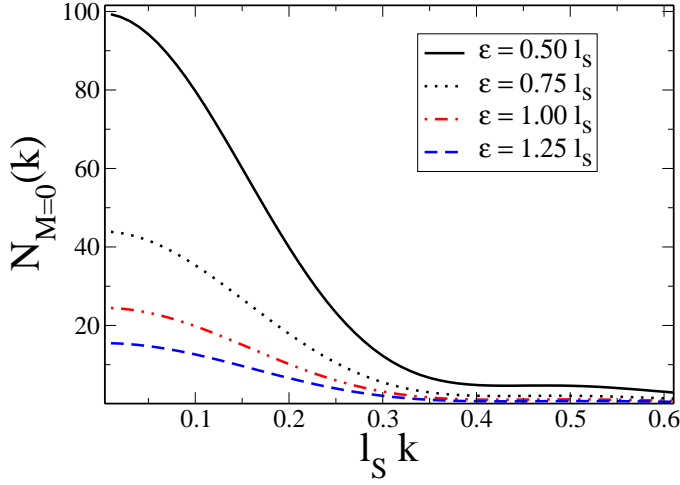


FIG. 4: $N_{M=0}(k)$ versus k for different values of ϵ . $R_0 = 10 l_s$ and $\eta = 1$.

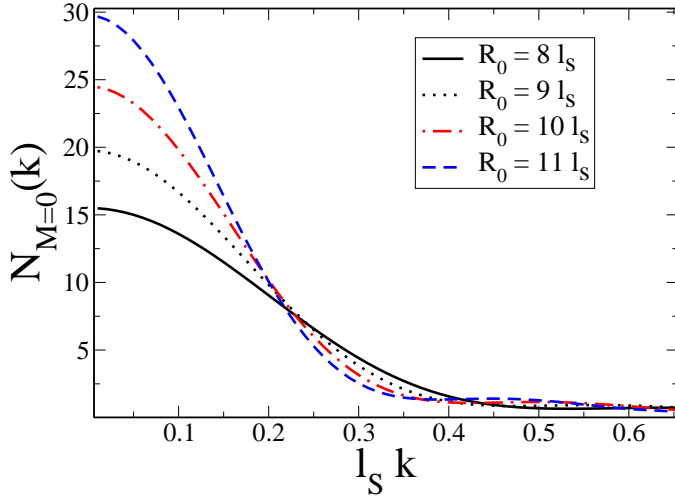


FIG. 5: $N_{M=0}(k)$ versus k for different values of R_0 . $\epsilon = l_s$ and $\eta = 1$.

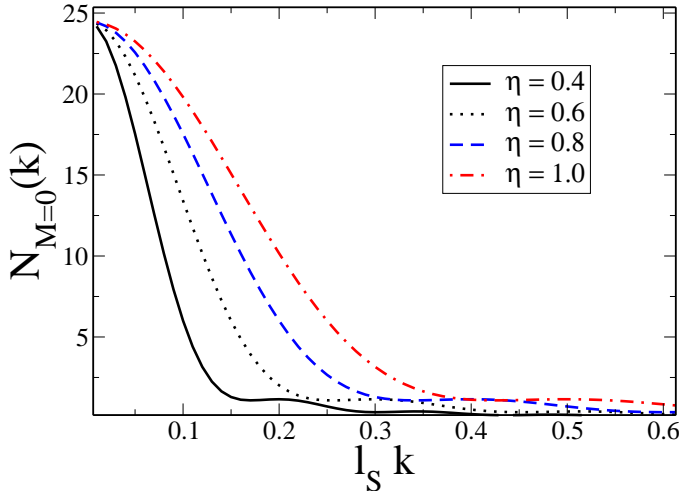


FIG. 6: $N_{M=0}(k)$ versus k for different values of η . $R_0 = 10 l_s$ and $\epsilon = l_s$.

For the massless modes we only integrate over k and sum over the 2 polarizations, since $N_{M>0}(k)$ has no dependence on the mode indices:

$$\rho_{M=0} = 2 \int \frac{d^3 k}{(2\pi)^3} k N_{M=0}(k). \quad (65)$$

As anticipated above, the integrals in (64), (65) are not convergent due to the fact that the simplified tachyon background (49) has discontinuous time derivatives at $t = 0$ and $t = t_c$. This has the consequence that the spectrum $N(k)$ decreases like k^{-2} for large k , which is too slow for convergence. In reality n th derivatives of $R(t)$ do not exceed $O(1/l_s^n)$, so the production of modes with $k > l_s^{-1}$ should be exponentially suppressed. We therefore introduce a UV cutoff, $k_{\max} = \Lambda \sim l_s^{-1}$. Moreover $N(k)$ is divergent as the vortex radius $\epsilon \rightarrow 0$, so ϵ is also presumably limited by the string scale, $\epsilon \sim l_s$.

In the final state, the heavy KK modes have mass given by c_{mn}/ϵ , where the c_{mn} 's are order unity and larger. These are near the cutoff, so their contributions are of the same order as other UV contributions which we are omitting. For consistency we should thus neglect the massive states' contribution to the total energy density on the vortex. Again, this underestimates the efficiency of particle production and makes our estimates conservative. Henceforth we will refer to $\rho_{M=0}$ as simply ρ .

Although the integral (65) can be performed analytically, the resulting expression for ρ is cumbersome. Rather than write it out explicitly we will discuss some noteworthy features, plot ρ with respect to the parameters of the model and present some useful simplifications of the complete expression in various limits.

Figures 7-8 show the dependence of ρ on the initial size of the brane. For R_0 greater than a few times l_s , the energy density is relatively insensitive to changes in R_0 . We can therefore reduce the dimensionality of the parameter space by simply assuming that the extra dimensions are somewhat larger than l_s . In fact in the limit of large R_0 , the energy density takes the very simple form

$$\lim_{R_0 \rightarrow \infty} \rho = \frac{\eta^2}{8\pi^2} \frac{\Lambda^2}{\epsilon^2} \quad (66)$$

which is the main result of this section. We recall that η , which parametrizes the speed at which the vortex forms, is predicted from eq. (46) to be $\eta = 1$.

As a check on our calculations, we have also considered the limit as $R_0 \rightarrow \epsilon$, which corresponds to a static background, with no vortex condensation. As expected, the energy density of produced particles goes to zero,

$$\rho \cong \frac{\Lambda^4}{4\pi^2 \epsilon^2} (R_0 - \epsilon)^2 - \frac{\Lambda^4}{4\pi^2 \epsilon^3} (R_0 - \epsilon)^3 + \dots$$

as $R_0 \rightarrow \epsilon$.

D. Efficiency of Reheating

To quantify the efficiency of the reheating we need to determine how much energy is available to produce the

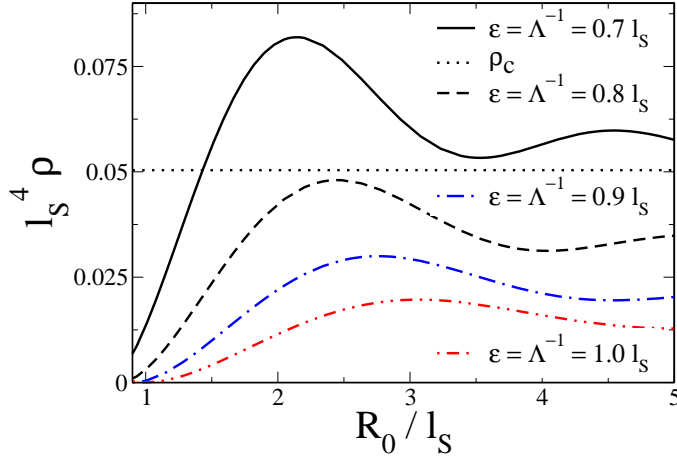


FIG. 7: ρ versus compactification radius R_0 for different values of $\epsilon = \Lambda^{-1}$ with $\eta = 1$. Dotted line is the critical density for efficient reheating ρ_c , (67).

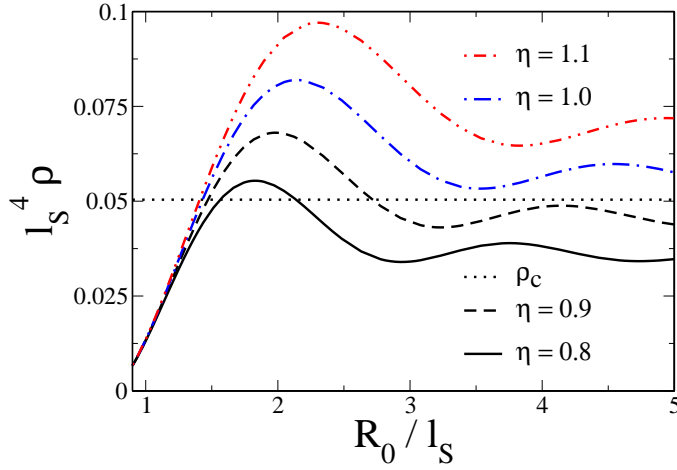


FIG. 8: Same as figure 7 but for different values of vortex collapse rate η and with $\epsilon = \Lambda^{-1} = 0.7 l_s$.

photons on the final-state 3-brane. Initially the system consisted of a D5-brane plus antibrane, whose 3D energy density was given by $2\tau_5 V_2$, where τ_5 is the tension of a D5-brane and V_2 is the volume of the compact 2-space $\{r, \theta\}$ wrapped by the branes. The final state consists of a D3-brane/antibrane with total tension $2\tau_3$. Until now we considered just half of the 2-sphere and focused on a single vortex located at $r = 0$. Conservation of Ramond-Ramond charge requires the second vortex, which we place at the south pole of the sphere to preserve azimuthal symmetry. These two defects are identical and are matched at the equator of the sphere. The vortex at the south pole represents the D3-antibrane.

Since reheating on each final-state brane should be equally efficient, the 3D energy density available for reheating on one them, which we call the critical energy density ρ_c , is half the difference between the initial and

final tensions of the branes and antibranes:

$$\rho_c \equiv \tau_5 V_2 - \tau_3 = \tau_3 \left(\frac{V_2}{4\pi^2 \alpha'} - 1 \right)$$

where we have used the recursion relation $2\pi\sqrt{\alpha'}\tau_p = \tau_{p-1}$. The tension of a D3-brane is given by [27]

$$\tau_3 = \frac{1}{g_s} \frac{1}{(2\pi)^3 \alpha'^2}.$$

The string coupling g_s , in 5+1 dimensions of which two are compact, is determined by the gauge coupling evaluated at the string scale, $\alpha(M_s)$ [2]:

$$g_s = \frac{V_2}{2\pi^2 \alpha'} \alpha(M_s).$$

Thus we have

$$l_s^4 \rho_c = \frac{1}{16\pi^3 \alpha(M_s)} \left(1 - \frac{4\pi^2 l_s^2}{V_2} \right). \quad (67)$$

In the regime where $V_2 \sim 2\pi R_0^2 \gg l_s^2$, which was where we could most easily quantify the particle production, the second term in parentheses can be neglected, and in any case it would be unimportant for a rough estimate unless it accidentally canceled the first term (1) to high accuracy. Hence we drop this term and take $l_s^4 \rho_c \cong (16\pi^3 \alpha(M_s))^{-1}$.

The energy density ρ_c is the critical value at which the conversion into radiation would be 100% efficient. In our analysis we take $\alpha(M_s) \cong \frac{1}{25}$ [2] which gives $\rho_c l_s^4 = 0.05$. The critical energy density is shown as a dashed horizontal line in figures 7 and 8. We see that the criterion for efficient reheating can be achieved for moderate values of the parameters. We only need for the length-scale cutoffs $1/\Lambda$ and ϵ to be somewhat smaller than the string scale, while the size of the extra dimensions should exceed a few times l_s . Using (66), we can write the criterion for efficient reheating as

$$\sqrt{\frac{\epsilon}{\eta \Lambda}} \lesssim (2\pi \alpha(M_s))^{1/4} l_s \cong 0.7 l_s \quad (68)$$

We have not taken into account the back-reaction of the particle production on the tachyon background, which is why our calculation allows for more reheating than is energetically possible. The back reaction will suppress somewhat the actual efficiency of reheating, but we don't expect a dramatic reduction. Given that we have been conservative in our estimates, such as ignoring the contributions from produced KK photons which will decay into massless photons, our result makes it plausible that a large fraction of the original energy can be converted into visible radiation.

VIII. SUMMARY AND CONCLUSION

We have argued for the possibility that our visible universe might be a codimension-two brane left over from

annihilation of a D5-brane/antibrane pair at the end of inflation. In this picture, reheating is due to production of standard model particles (*e.g.* photons) on the final branes, driven by their couplings to the tachyon field which encodes the instability of the initial state as well as the vortex which represents the final brane. We find that reheating can be efficient, in the sense that a sizable fraction of the energy available from the unstable vacuum can be converted into visible radiation, and not just gravitons.

The efficiency of reheating is greatest if the radius of compactification of the extra dimensions is larger than 2-3 times the string length l_s . The efficiency also depends on phenomenological parameters we had to introduce by hand in order to cut off ultraviolet divergences in the calculated particle production rate: namely ϵ , a nonvanishing radius for the final brane, and Λ , an explicit cutoff on the momentum of the photons produced. The latter must be introduced to correct for discontinuous time derivatives in our simplified model of the background tachyon condensate; the actual behavior of the condensate corresponds to a cutoff of order $\Lambda \sim 1/l_s$. It is less obvious why the effective field theory treatment should give divergent results as the thickness of the final brane (2ϵ) goes to zero, but it seems clear that a fully string-theoretic computation would give no such divergence, and therefore it is reasonable to cut off the field-theory divergence at $\epsilon \sim l_s$. Given only these mild assumptions, our estimates (66,67) predict that the fraction of the available energy which is converted into visible radiation is $\rho/\rho_c \cong \pi\alpha(M_s) \cong 0.25$. This simple estimate counts only photons; in a more realistic calculation, it would be enhanced by the number of light degrees of freedom which couple to the tachyon, which could be much greater than 1. Moreover it could also be enhanced by the production of massive KK modes, which correspond to string excitations when the vortex has formed [25, 28].

Our analysis makes significant improvements to the previous work of [5]. We considered the formation of a vortex in the tachyon field rather than a kink, in line with the descent relations for stable Dp-branes. We found analytic solutions for the tachyon field which give the time dependence in the vicinity of the defect while it is forming, for both the vortex and the kink solutions. In the latter case we verified that this solution reproduces the known dynamics of kink formation which was determined numerically in [10], giving us more confidence in the vortex solutions, which are quite analogous. Our explicit solution for the tachyon background is nevertheless too complicated for computing the production of particles on the defect. We therefore approximated it by a simpler ansatz with the same qualitative behavior, which allows for analytic solutions of the gauge fields in the background. This ansatz resembles a gauge field in a 6D spacetime with two compact spatial dimensions which are contracting with time, and leads to simple analytic results for the energy density of photons produced during the contraction, in the regime where the extra dimensions

are large compared to the string scale.

There are still some outstanding questions to be addressed concerning this scenario. First, we have made reference to the Kibble mechanism for the creation of the final state defects. If we assume the causal bound of one defect per Hubble volume then this would imply that the size of the extra dimensions must exceed the inverse Hubble rate; otherwise there would be enough time for the fields to straighten themselves out and the putative vortex-antivortex pair would immediately annihilate. For example if we take the string scale M_s to be 10^{16} GeV then $H \sim M_s^2/M_p$ and we would need the compactification scale to be of order $R_0 \sim (M_p/M_s)l_s$. Our results indicate that efficient reheating is compatible with a large compactification scale. However, taking the remaining four extra dimensions (which the initial 5-brane/antibrane pair do not wrap) to be string scale is not consistent with getting inflation since the initial state 5-brane and antibrane cannot be sufficiently separated to satisfy the slow roll conditions.

On the other hand, our results indicate that the reheating *can* be efficient for R_0 only a few times the string length, though in this scenario a naive application of the Kibble mechanism does not favor having the final state defects span the three large dimensions. These requirements may not be prohibitive since the question of how these defects form dynamically at the end of inflation is a quantitative one which merits further investigation. In principle the correlation length for the initial fluctuations of the tachyon field could be as small as the string length. We point out also that it is possible that the dynamics of the formation of tachyon defects is qualitatively different from defect formation in a conventional scalar field theory. For example, the numerical investigation of [10], it was found that small kinks in the initial configuration which are in causal contact with each other *do not* dynamically straighten themselves out as they would in a conventional, nontachyonic field theory. Instead, every place where the field crosses zero in the initial state develops a full-blown kink, so long as there was enough energy in the bulk to produce the required number of kinks. Another indication that the dynamics of the tachyon field may be qualitatively different from an ordinary scalar field theory comes from the [29] in which the causal structure of the tachyon Dirac-Born-Infeld action was studied. The authors of [29] found that small fluctuations of the tachyon field propagate according to an effective metric which depends on the tachyon background. In the case of a homogeneous rolling background it was found that as the condensation proceeds the effective metric contracts to the Carroll limit of the Lorentz group so that the tachyon light cone collapses into a timelike half line and the tachyon fields at different spatial points are decoupled. We feel that quantitatively determining the dynamics of the formation of tachyon defects at the endpoint of D-brane inflation is a question which deserves further investigation.

In the present analysis we have not included the grav-

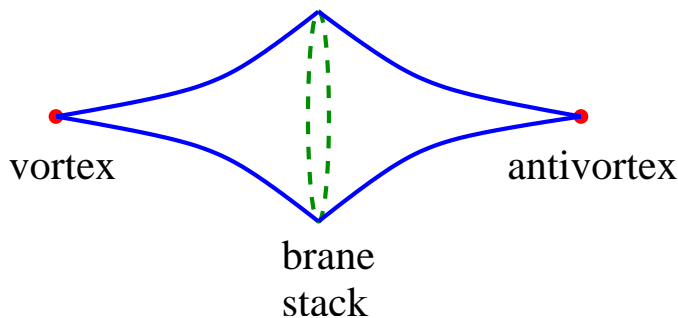


FIG. 9: Warped compactification with branes localized in throats on opposite sides of a stack of branes.

itational or Ramond-Ramond forces between the vortex and antivortex which would attract them toward each other and lead to their eventual annihilation. How do we insure that the braneworld on which we are supposed to live is safe from annihilation with an antibrane in the bulk? One possibility is to have warping caused by a stack of branes which wraps only the equator of the extra dimensions, as illustrated in fig. 9. Such a braneworld scenario using the AdS soliton solution for the bulk has been considered in ref. [7, 30, 31]. The advantage for our scenario is that the warping can provide a barrier to the annihilation of the brane-antibrane pair, since it is

energetically favorable for them to remain within their respective throats.

In solving for the tachyon background we have also ignored the possibility of caustic formation in the bulk [17] by taking initial profiles without too much curvature. It is possible that caustic formation may be an artifact of the derivative truncation which leads to the Born-Infeld type of Lagrangian for the tachyon. See [32] for a discussion of the problems of dynamical equations with infinitely many derivatives.

Brane-antibrane inflation and braneworld cosmology are two of the most important applications of string theoretic ideas to cosmology. We find it intriguing that these two ideas might be combined in the way we have described. An outstanding challenge is to find some observable signatures that would be able to test our scenario, for example through the gravitational wave component which is expected to be a major component of the radiation produced during reheating.

We thank Horace Stoica for helpful discussions concerning warped compactifications as well as Robert Brandenberger and Henry Tye for helpful comments regarding the Kibble mechanism. Our research is supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada and Fonds québécois de la recherche sur la nature et les technologies (NATEQ).

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